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A Simple Procedure To Amortize A Loan

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ABSTRACT

This note offers a very simple procedure to amortize a loan. This procedure is based on a very basic property that the present value today of the amount of principal repaid in a level periodic repayments on a fixed rate of interest loan in any period remains the same. The traditional approach to maintain continuity has been described in section I, section II provides alternative simple procedure for amortization, and section III concludes with summary

AS TEACHERS IN finance we have all taught the construction of amortization schedule to our students. Usually it involves the stepwise construction of a table, called Amortization Schedule. Eventhough the procedure is not difficult, but is very time consuming. Furthermore, if one wants to know the amount of principal paid at, say, point t in a n-period loan, the procedure involves calculating the loan balance remaining at t and then subtracting the interest rate times the loan balance remaining at time t from the level periodic payments. Any standard textbook such as Prakash, Karels and Fernandez [1] provides the formula for the computation of loan balance remaining. In this paper, we provide an alternative and simple procedure to construct an amortization schedule that involves the computation of level payments, the principal and the interest paid in only the first period. In section I, we briefly describe the traditional approach to maintain continuity. In the second section, we provide the alternative simple procedure for amortization. The last section concludes with a summary.

The Traditional Approach

We will use the following notation:

A = the amount of loan at time 0
BR_t = the loan balance remaining at time t
n = term of the loan
i = periodic rate of interest on the loan
PP_t = amount of principal repaid at time t

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IE_t = interest expense at time t
L = level of payments required at the end of each period to pay off the loan

Prakash, Kavels and Fernandez (1987) show that

$$L = A \left[\frac{i}{1 - (1+i)^{-1}} \right] \qquad \textbf{(1)}$$

$$BR_t = A \left[\frac{1-(1+i)^{-(n-t)}}{1-(1+i)^{-n}} \right] \qquad \textbf{(2)}$$

Using (1) the typical amortization schedule looks like Table 1 below

Table 1

Amortization Schedule for a Loan Amount A

Period	Level Payment*	Interest Expense	Principal Repaid	Balance Remaining
1	L	IE_1 =iA	PP_1 = L-iA	BR_1 = A - PP_1
2	L	IE_2 = I(BR_1)	PP_2 = L-i(BR_1)	BR_2= BR_1 - PP_2
3	L	IE_3 = i(BR_2)	PP_3 = L-i(BR_2)	BR_3= BR_2 - PP_3
.	.	.	.	
.	.	.	.	
.	.	.	.	
n	L	IE_n=i(BR_{n-1})	PP_n = L-i(BR_{n-1})	BR_n=BR_{n-1}-PP_n= 0

* computed using expression (1)

From Table 1, it is clear that in order to obtain the principal repaid or interest expense at t, the balance remaining at t- 1 must be computed either by stepwise procedure described in the table or using equation (2).

II. AN ALTERNATIVE SIMPLE APPROACH

The construction of the alternative amortization schedule is based on a simple property.

That is, the principal repaid at t, i.e., PP, is equal to

$PP_t = PP_{t-1}\,(1+i) = PP_1\,(I+i)^{t-1}$

To prove this, we have

$PP_t = L - i(BR_{t-1})$, and

$PP_{t-l} = L - i(BR_{t-2})$

or $\quad PP_t + i(BR_{t-1}) = PP_{t-1} + i(BR_{t-2}) \qquad \textbf{(3)}$

$$PP_t = PP_{t-1} + i[BR_{t-2} - BR_{t-1}]$$

But in expression (3) $BR_{t-2} - BR_{t-1} = PP_{t-l}$

Thus (3) become,

$$PP_t = PP_{t-l} + iPP_{t-1} \qquad \textbf{(4)}$$

or $PP_t = PP_{t-1}(1+i)$

Since (4) is valid for any t we have recursively

$$PP_2 = PP_l(1+i)$$
$$PP_3 = PP_2(1+i) = PP_l(l+i)^2$$
$$PP_4 = PP_3(1+i) = PP_l(l+i)^3$$
$$. \quad . \quad .$$
$$. \quad . \quad .$$
$$. \quad . \quad . \qquad \textbf{(5)}$$
$$Pp_n = PP_{n-l}(1+i) = PP_1(l+i)^{n-I}$$

Using the results obtained in (5), the amortization schedule can be easily computed as follows:

Step 1: Compute L using expression (1). Fill in the first column of Table 2 with this amount.
Step 2: Compute $PP_1 = A - iL$
Compute $PP_{2,}$ - - - , PP_n using the above result.
i.e. $PP_t = PP_{t-1},(l+i) = PP_1(l+i)^{t-1}$
Step 3: Compute interest expense by subtracting column (ii) from column (i).
Step 4: Commute loan balance remaining as indicated in column (iv).

Table 2

Amortization Schedule for a Loan Amount A

Period	Level Payment (i)	Principal Repaid (ii)	Interest Expense (iii)	Balance Remaining (iv)
1	L	PP_1 =L-iA	IE_1 =L-PP_1	BR_1 = A-- PP_1
2	L	PP_2=$PP_l(1+i)$	IE_2 =L-PP_2	BR_2=BR_1- PP_2
3	L	PP_3 =$PP_l(l+i)^2$	IE_3 =L-PP_3	BR_3=BR_2- PP_3
.	.	.		
.	.	.		
.	.	.		
n	L	PP_n =$PP_l(l+i)^{n-1}$	IE_n =L-PP_n	BR_n=BR_{n-1-}-PP_n=0

Note that in this method the computation of principal repaid in any period is independent of balance remaining in the previous period, making this construction of schedule easier than the usual method.

Summary & Conclusion

The method of amortization presented in this paper is quick and easy to understand. Essentially, it entails the computation of L and the rest of the computations are straight forward. Also note that this result admits to a very interesting concept. That is, the present value today of the principal repaid in any period remains constant. For, we have

$$PP_t = PP_1(1+i)^{t-1} = PP_1(1+i)^{-1}(1+i)^t = k(1+i)^t$$

where k is the present value of PP_1 at point zero and equals $PP_t|(1+i)^t$ for any t, hence the present value today of the principal repaid in any period is constant.

References

1. Arun Prakash, Gordon V. Karels and Ray Fernandez., *Financial Commercial and Mortgage Mathematics and Their Applications* (New York: Praeger, 1987).